

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z \quad (\text{A-1})$$

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \quad (\text{A-2})$$

$$(\mathbf{A} \times \mathbf{B}) \times \mathbf{C} = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{B} \cdot \mathbf{C})\mathbf{A} \quad (\text{A-3})$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C} \quad (\text{A-4})$$

$$(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{A} \cdot \mathbf{D})(\mathbf{B} \cdot \mathbf{C}) \quad (\text{A-5})$$

$$\begin{aligned} (\mathbf{A} \times \mathbf{B}) \times (\mathbf{C} \times \mathbf{D}) &= [\mathbf{A} \cdot (\mathbf{B} \times \mathbf{D})]\mathbf{C} - [\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})]\mathbf{D} \\ &= [\mathbf{A} \cdot (\mathbf{C} \cdot \mathbf{D})]\mathbf{B} - [\mathbf{B} \cdot (\mathbf{C} \times \mathbf{D})]\mathbf{A} \end{aligned} \quad (\text{A-6})$$

$$\nabla \phi \psi = \phi \nabla \psi + \psi \nabla \phi \quad (\text{A-7})$$

$$\nabla \cdot (\phi \mathbf{a}) = \phi \nabla \cdot \mathbf{a} + (\nabla \phi) \cdot \mathbf{a} \quad (\text{A-8})$$

$$\nabla \times (\phi \mathbf{a}) = \phi (\nabla \times \mathbf{a}) + \nabla \phi \times \mathbf{a} \quad (\text{A-9})$$

$$\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot (\nabla \times \mathbf{a}) - \mathbf{a} \cdot (\nabla \times \mathbf{b}) \quad (\text{A-10})$$

$$\nabla \times (\mathbf{a} \times \mathbf{b}) = \mathbf{a}(\nabla \cdot \mathbf{b}) + (\mathbf{b} \cdot \nabla)\mathbf{a} - \mathbf{b}(\nabla \cdot \mathbf{a}) - (\mathbf{a} \cdot \nabla)\mathbf{b} \quad (\text{A-11})$$

$$\nabla(\mathbf{a} \cdot \mathbf{b}) = (\mathbf{a} \cdot \nabla)\mathbf{b} + (\mathbf{b} \cdot \nabla)\mathbf{a} + \mathbf{a} \times (\nabla \times \mathbf{b}) + \mathbf{b} \times (\nabla \times \mathbf{a}) \quad (\text{A-12})$$

$$\nabla \times (\nabla \times \mathbf{a}) = \nabla(\nabla \cdot \mathbf{a}) - \nabla^2 \mathbf{a} \quad (\text{A-13})$$

$$\nabla \cdot (\nabla \times \mathbf{a}) = 0 \quad (\text{A-14})$$

$$\nabla \times (\nabla \phi) = 0 \quad (\text{A-15})$$

The line element in orthogonal coordinates is

$$ds^2 = (h_1 dx_1)^2 + (h_2 dx_2)^2 + (h_3 dx_3)^2$$

The ∇ operations are

$$\nabla \psi = \frac{1}{h_1} \frac{\partial \psi}{\partial x_1} \mathbf{e}_1 + \frac{1}{h_2} \frac{\partial \psi}{\partial x_2} \mathbf{e}_2 + \frac{1}{h_3} \frac{\partial \psi}{\partial x_3} \mathbf{e}_3$$

$$\nabla \cdot \mathbf{a} = \frac{1}{h_1 h_2 h_3} \left[\frac{\partial}{\partial x_1} (h_2 h_3 a_1) + \frac{\partial}{\partial x_2} (h_1 h_3 a_2) + \frac{\partial}{\partial x_3} (h_1 h_2 a_3) \right]$$

$$\begin{aligned} \nabla \times \mathbf{a} &= \frac{1}{h_2 h_3} \left[\frac{\partial}{\partial x_2} (h_3 a_3) - \frac{\partial}{\partial x_3} (h_2 a_2) \right] \mathbf{e}_1 \\ &\quad + \frac{1}{h_1 h_3} \left[\frac{\partial}{\partial x_3} (h_1 a_1) - \frac{\partial}{\partial x_1} (h_3 a_3) \right] \mathbf{e}_2 \\ &\quad + \frac{1}{h_1 h_2} \left[\frac{\partial}{\partial x_1} (h_2 a_2) - \frac{\partial}{\partial x_2} (h_1 a_1) \right] \mathbf{e}_3 \end{aligned}$$

$$\nabla^2 \psi = \frac{1}{h_1 h_2 h_3} \left[\frac{\partial}{\partial x_1} \left(\frac{h_2 h_3}{h_1} \frac{\partial \psi}{\partial x_1} \right) + \frac{\partial}{\partial x_2} \left(\frac{h_3 h_1}{h_2} \frac{\partial \psi}{\partial x_2} \right) + \frac{\partial}{\partial x_3} \left(\frac{h_1 h_2}{h_3} \frac{\partial \psi}{\partial x_3} \right) \right]$$