Meaning of ionospheric Joule heating

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[1] The possibility of relating the electric current in the ionosphere to the electric field in the frame of reference either of the neutral atmosphere (as commonly done) or of the plasma raises the question: which should be used in calculating Joule heating? The energy equations for the plasma and for the neutral medium, including collision effects, can be combined with momentum equations to separate energy transfer into work done and heating (this is *not* the same as separation of energy content into bulk-flow kinetic energy and thermal energy). Heating/dissipation processes can be unambiguously identified, showing that true electromagnetic (Joule) heating rate is given by $\mathbf{J} \cdot \mathbf{E}$ in the frame of reference of the plasma. The major part of energy dissipation associated with $\mathbf{J}$ and $\mathbf{E}$ in the ionosphere is heating by collisions between plasma and neutrals, proportional to the square of the difference in their bulk flow velocities and distributed approximately equally between the two. The conventionally calculated ionospheric Joule heating as $\mathbf{J} \cdot \mathbf{E}$ in the frame of reference of the neutral atmosphere equals the sum of total heating rate of plasma (only a small part of which is true Joule heating) plus total heating rate of neutrals, plus a term equal to work done on plasma in the neutral-atmosphere frame of reference; the latter is assumed negligible, with $\mathbf{J} \times \mathbf{B}/c$ balanced entirely by plasma-neutral collisions and with no net work done on the plasma. This assumption implies that all the magnetic stresses exerted on the plasma at ionospheric heights are taken up by the local neutral medium; hence the dynamics of the neutral atmosphere play an important role in magnetosphere/ionosphere interactions.

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1. Introduction

[2] In studies of energy input to the ionosphere [e.g., *Lu et al.*, 1995; *Thayer et al.*, 1995; *Fujii et al.*, 1999; *Thayer*, 2000; *Richmond and Thayer*, 2000], the rate of energy conversion from electromagnetic to mechanical form, $\mathbf{J} \cdot \mathbf{E}$, is commonly expressed as the sum of two terms, Joule heating and work done. Defining Joule heating in this context, however, is not as simple as might seem at first. In a strict sense, Joule or Ohmic heating is that given by ηJ^2 where η is the resistivity, and it appears as part of $\mathbf{J} \cdot \mathbf{E}$ whenever there is a linear relation between $\mathbf{J}$ and $\mathbf{E}$. Because $\mathbf{E}$ varies with frame of reference, while $\mathbf{J}$ in a nonrelativistic, quasi-neutral plasma does not, $\mathbf{J}$ cannot depend on $\mathbf{E}$ simply but only on

$$\mathbf{E}^* = \mathbf{E} + \mathbf{V} \times \mathbf{B}/c, \tag{1}$$

the electric field in a particular frame of reference. Historically, Ohm's law was first established as an observed proportionality of current to electric field in a material medium; in this case, obviously, the velocity $\mathbf{V}$ must be that of the medium, and Joule or Ohmic heating can be uniquely identified with $\mathbf{J} \cdot \mathbf{E}$ in the frame of reference in which the medium is at rest. In the ionosphere, however, there are two media, the plasma with bulk velocity $\mathbf{U}$ and the neutral atmosphere with $\mathbf{u_n}$. Conventionally, the ionospheric Ohm's law is expressed in terms of $\mathbf{E}^*$ with $\mathbf{V} = \mathbf{u_n}$, but *Song et al.* [2001] have shown that on the basis of the *same* physical assumptions, $\mathbf{J}$ can be written as proportional both to $\mathbf{E} + \mathbf{U} \times \mathbf{B}/c$ and to $\mathbf{E} + \mathbf{u_n} \times \mathbf{B}/c$, with *different* conductivity coefficients; this is possible because $\mathbf{U} - \mathbf{u_n}$ is proportional to $\mathbf{J}$, by virtue of the plasma momentum equation. In fact, it is easily shown that, with appropriate redefinition of the conductivity coefficients, $\mathbf{J}$ can be rewritten in terms of $\mathbf{E} + \mathbf{V} \times \mathbf{B}/c$ with *any* $\mathbf{V}$ that satisfies

$$\mathbf{V} = \mathbf{U} + \zeta(\mathbf{u_n} - \mathbf{U}), \tag{2}$$

where ζ is an arbitrary constant. If Joule heating is taken as $\mathbf{J} \cdot \mathbf{E}^*$, what $\mathbf{V}$ should be used in equation (1)?

[3] In this paper we discuss the physical meaning to be attached to the various definitions of Joule heating in the

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ionosphere, on the basis both of equations relating **J** and **E** and of the energy equations of plasma and neutrals. We note that the electromagnetic fields interact directly only with the plasma and that any effects on neutrals, including energy transfer, can only occur by means of plasma-neutral collisions. We examine the common assumption that Joule heating represents the power going into thermal energy (whether of the plasma or of the neutral gas is not always clear) and that the work done represents the mechanical energy transfer into the neutral gas, with the tacit (and sometimes [*Richmond and Thayer*, 2000] explicit) understanding that it goes into kinetic energy of bulk flow of the neutral gas.

2. Equations Relating J and E in the Ionosphere

[4] Because all charged particles contribute to it, **J** appears in only those plasma equations that are derived from the velocity moments of the Boltzmann equations summed over all species. (**E**, by contrast, can appear also in equations referring to individual species or even to single particles.) Particularly important is the generalized Ohm's law, specified precisely as the velocity moment equation that gives $\partial\mathbf{J}/\partial t$ [see, e.g., *Rossi and Olbert*, 1970; *Greene*, 1973]:

$$\partial\mathbf{J}/\partial t = \sum_a \left\{ \left(q_a^2 n_a/m_a\right)(\mathbf{E} + \mathbf{V_a} \times \mathbf{B}/c) - (q_a/m_a)\nabla\cdot\boldsymbol{\kappa}_a + q_a n_a \mathbf{g} \right\} + (\delta\mathbf{J}/\delta t)_{coll} \quad (3)$$

where q_a, m_a, n_a, $\mathbf{V_a}$, and $\boldsymbol{\kappa}_a$ are the charge, mass, concentration, bulk velocity, and kinetic tensor, respectively, of charged particles of species a, and $(\delta\mathbf{J}/\delta t)_{coll}$ represents the sum of all collision effects. The momentum equation for the entire plasma (subsuming into the plasma medium all the charged particles but not the neutrals) is

$$\partial\rho\mathbf{U}/\partial t + \nabla\cdot\boldsymbol{\kappa} = \mathbf{J}\times\mathbf{B}/c + (\delta\rho\mathbf{U}/\delta t)_{pn}, \quad (4)$$

where ρ is the mass density, **U** is the bulk velocity, and $\boldsymbol{\kappa} = \sum_a \boldsymbol{\kappa}_a$ (more commonly written as $\boldsymbol{\kappa} = \rho\mathbf{UU} + \mathsf{P}$); **E** does not appear because of quasi-neutrality. (In equation (4) we have omitted gravity and inertial (e.g., Coriolis) force terms which play no significant role in our discussion.)

[5] The collision term in equation (4) is labeled *pn* to emphasize that only collisions of plasma with neutral particles contribute to the momentum equation. Given the electron-ion, electron-neutral, and ion-neutral collision frequencies ν, the collision terms are

$$(\delta\mathbf{J}/\delta t)_{coll} = -(\nu_{ei} + \nu_{en} + (m_e/m_i)\nu_{in})\mathbf{J} + (\nu_{en} - \nu_{in})n_e e(\mathbf{U} - \mathbf{u_n}) \quad (5)$$

[see, e.g., *Song et al.*, 2004] and

$$(\delta\rho\mathbf{U}/\delta t)_{pn} = -n_e(m_i\nu_{in} + m_e\nu_{en})(\mathbf{U} - \mathbf{u_n}) - (m_e/e)(\nu_{in} - \nu_{en})\mathbf{J} \quad (6)$$

[e.g., *Song et al.*, 2001]; for later use, we will simplify equation (6) by neglecting the **J** term (of order m_e/m_i) and defining a single effective plasma-neutral collision frequency to write

$$(\delta\rho\mathbf{U}/\delta t)_{pn} = \nu_{pn}\rho(\mathbf{u}_n - \mathbf{U}). \quad (7)$$

[6] Equation (3) is exact as it stands. The left-hand side, however, is negligible whenever only time scales $\gg 1/\omega_p$ and spatial scales $\gg \lambda_e = c/\omega_p$, where ω_p is the (electron) plasma frequency, are considered [*Vasyliūnas*, 1996]. The gravity term $\rho_c\mathbf{g}$ (ρ_c = charge density) is practically always negligible, and the kinetic tensor terms usually are neglected in the ionosphere. With these approximations and with the further assumption of electrons and one species of ions, $m_e/m_i \ll 1$, the generalized Ohm's law (divided by $n_e e^2/m_e$ for convenience) simplifies to

$$0 = \mathbf{E} + \mathbf{U}\times\mathbf{B}/c - \mathbf{J}\times\mathbf{B}/nec + \left(m_e/n_e e^2\right)(\delta\mathbf{J}/\delta t)_{coll}. \quad (8)$$

[7] A resistive term $\eta\mathbf{J}$ arises from any part of $(\delta\mathbf{J}/\delta t)_{coll}$ proportional to **J**. From equation (5) the resistivity coefficient η is

$$\eta = m_e(\nu_{ei} + \nu_{en} + (m_e/m_i)\nu_{in})/n_e e^2. \quad (9)$$

The dominant resistive effect comes from electron collisions, with ion-neutral collisions contributing only a correction term of order m_e/m_i. The $\mathbf{U} - \mathbf{u_n}$ term in equation (5) likewise contributes only a term of order m_e/m_i to equation (8); this can be shown from the relation between $\mathbf{U} - \mathbf{u_n}$ and **J** that follows from equation (6) and the momentum equation (4) (in which the time-derivative and kinetic-tensor terms are assumed to be negligible). Leaving out these small corrections, we have

$$0 = \mathbf{E} + \mathbf{U}\times\mathbf{B}/c - \mathbf{J}\times\mathbf{B}/nec - \eta\mathbf{J} \quad (10)$$

with

$$\eta = m_e(\nu_{ei} + \nu_{en})/n_e e^2. \quad (11)$$

The Joule heating ηJ^2 is equal to $\mathbf{J}\cdot(\mathbf{E} + \mathbf{U}\times\mathbf{B}/c)$; it becomes negligible if electron collision effects are negligible.

[8] By contrast, ionospheric Joule heating as conventionally defined is taken to be equal to $\mathbf{J}\cdot(\mathbf{E} + \mathbf{u_n}\times\mathbf{B}/c)$. Over much of the ionosphere, it depends primarily on ion-neutral collisions and is not greatly changed if electron collisions are neglected; only at altitudes below about 100 km do the electron collisions become unavoidably important. It can be derived from equation (8) if **U** everywhere (including the $\mathbf{U}\times\mathbf{B}/c$ term) is eliminated by using the momentum equation (4) (again with neglect of the time-derivative and kinetic-tensor terms): the resulting equation has the same form as equation (10), with **U** replaced by $\mathbf{u_n}$, a more complicated coefficient of $\mathbf{J}\times\mathbf{B}/nec$, and a different expression for η. However, while the mathematical form is the same, the physical meaning is not: $\eta\mathbf{J}$ in this version does not arise from balancing the collisional effects on **J** against $\mathbf{E}^*$ in the generalized Ohm's law but instead from balancing the collisional effects on $\mathbf{U} - \mathbf{u_n}$ against $\mathbf{J}\times\mathbf{B}/c$

in the plasma momentum equation (which, as noted before, does not contain **E**). The quantity $\mathbf{J} \cdot (\mathbf{E} + \mathbf{u_n} \times \mathbf{B}/c)$ would thus seem to be more accurately described not as Joule heating but as frictional heating due to the relative motion of plasma and neutrals.

3. Energy Equations

[9] Of course, the mere name "Joule heating" is not important. What matters is which expression most closely represents what would normally be called "heating" in the sense of thermal energy input into the medium. To decide this, it is necessary to examine the energy conservation equations of both plasma and neutrals. Generalizing the derivation for a fully ionized plasma presented by *Rossi and Olbert* [1970] to include plasma-neutral collisions (under the assumption that these are nonionizing and thus do not change the densities) gives the energy conservation equations:

$$\frac{\partial}{\partial t}\left[\frac{1}{2}\rho U^2 + \epsilon\right] + \nabla \cdot \left[\mathbf{U}\left(\frac{1}{2}\rho U^2 + \epsilon\right) + \mathsf{P} \cdot \mathbf{U}\right] = \mathbf{E} \cdot \mathbf{J} + \left(\frac{\delta W}{\delta t}\right)_{pn} - \nabla \cdot \mathbf{q} \tag{12}$$

for plasma, and

$$\frac{\partial}{\partial t}\left[\frac{1}{2}\rho_n u_n^2 + \epsilon_n\right] + \nabla \cdot \left[\mathbf{u}_n\left(\frac{1}{2}\rho_n u_n^2 + \epsilon_n\right) + \mathsf{P}_n \cdot \mathbf{u}_n\right] = \left(\frac{\delta W_n}{\delta t}\right)_{np} - \nabla \cdot \mathbf{q}_n \tag{13}$$

for the neutral medium. Here $\mathbf{q}$ and $\mathbf{q}_n$ are the heat flux vectors (which we neglect henceforth). The quantity ϵ is the thermal energy density, equal to (1/2)(Trace P) in general or (3/2)P for isotropic pressure, similarly, ϵ_n in terms of P_n. Any viscous effects appear as nonisotropic terms of P or P_n [e.g., *Landau and Lifshitz*, 1959]; see also the discussion following equation (39). The collision terms on the right-hand sides are related, by conservation of energy in collisions, as

$$(\delta W/\delta t)_{pn} = -(\delta W_n/\delta t)_{np}. \tag{14}$$

(By assuming the validity of equation (14), we are neglecting transfer of energy from kinetic energy of particle motion to any internal degrees of freedom that the particles might have, particularly in the case of molecular species of ions or neutrals. To first approximation, such effects can be included by adding a term to ϵ or ϵ_n to represent the additional energy density; a proper derivation, however, requires generalizing the concept of particle distribution function beyond the usual velocity space as well as treating collisions in adequate detail.)

[10] The left-hand sides of equations (12) and (13) are in standard conservation form: partial time derivative of density of, plus divergence of flux density of, the quantity that is conserved if the right-hand side equals zero. The conserved quantity here is total mechanical energy, of the plasma or of the neutral medium, respectively: kinetic energy of bulk flow plus thermal energy (or, in thermodynamic language, internal energy), the latter being kinetic energy of motion relative to bulk flow. The terms on the right-hand side represent the rate of change of total mechanical energy density by electromagnetic and collisional processes (the heat-flux terms, to be neglected anyway, should strictly speaking be on the left-hand side but have been placed on the right, for later convenience). Under change of frame of reference, equations (12) and (13) are covariant: their form remains the same but the values of the individual terms may change.

[11] Taking, separately for plasma and for neutrals, the dot product of the momentum equation with the bulk velocity gives an equation which can be recast in conservation form for the bulk flow kinetic energy density:

$$\frac{\partial}{\partial t}\left[\frac{1}{2}\rho U^2\right] + \nabla \cdot \left[\mathbf{U}\left(\frac{1}{2}\rho U^2\right)\right] + [\nabla \cdot \mathsf{P}] \cdot \mathbf{U} = -\mathbf{J} \cdot \left[\frac{\mathbf{U}}{c} \times \mathbf{B}\right] + \mathbf{U} \cdot \left(\frac{\delta \rho \mathbf{U}}{\delta t}\right)_{pn} \tag{15}$$

for plasma, and

$$\frac{\partial}{\partial t}\left[\frac{1}{2}\rho_n u_n^2\right] + \nabla \cdot \left[\mathbf{u}_n\left(\frac{1}{2}\rho u_n^2\right)\right] + [\nabla \cdot \mathsf{P}_n] \cdot \mathbf{u}_n = \mathbf{u}_n \cdot \left(\frac{\delta \rho_n \mathbf{u}_n}{\delta t}\right)_{np} \tag{16}$$

for neutrals; by conservation of momentum in collisions

$$(\delta \rho \mathbf{U}/\delta t)_{pn} = -(\delta \rho_n \mathbf{u}_n/\delta t)_{np}. \tag{17}$$

The right-hand sides of equations (15) and (16) represent the work done by the electromagnetic and collisional forces acting on the bulk flows. The rate of change of bulk flow kinetic energy density is determined, however, not only by work done by these forces (right-hand side) but also by work done against pressure (terms on the left). Whether work done by electromagnetic or collisional stresses contributes to changing the bulk flow or the thermal energy depends entirely on whether the stresses are balanced by the inertial or by the pressure terms in the momentum equation. Under change of frame of reference, equations (15) and (16) as written do not transform in any well-defined way and do not preserve their form.

[12] One can now eliminate the kinetic energy of bulk flow from the energy equation by subtracting equations (15) and (16) from equations (12) and (13). The right-hand sides of the resulting equations contain the electromagnetic and collisional energy input *minus* work-done terms. The left-hand sides can be transformed into a form that is invariant under change of frame of reference, through extensive manipulation (invoking also the continuity equation) given in detail by *Rossi and Olbert* [1970, pp. 293–295]. The results are: for plasma

$$\frac{3}{2}P\frac{d}{dt}\left[\log\left(\frac{P}{\rho^{5/3}}\right)\right] = \mathbf{J}^* \cdot \left[\mathbf{E} + \frac{\mathbf{U}}{c} \times \mathbf{B}\right] + \left(\frac{\delta Q}{\delta t}\right)_{pn}, \tag{18}$$

same as equation (10.147) of *Rossi and Olbert* [1970] except for the addition of plasma-neutral collision terms (and neglect of heat flux and viscous heating terms—it is at this point that they must be put on the right-hand side), and for neutrals

$$\frac{3}{2}P_n\left(\frac{d}{dt}\right)_n\left[\log\left(\frac{P_n}{\rho_n{}^{5/3}}\right)\right]=\left(\frac{\delta Q_n}{\delta t}\right)_{np}. \qquad (19)$$

The collision terms $(\delta Q/\delta t)$ are defined by

$$(\delta Q/\delta t)_{pn}=(\delta W/\delta t)_{pn}-\mathbf{U}\cdot(\delta\rho\mathbf{U}/\delta t)_{pn} \qquad (20)$$

for plasma, and

$$\begin{aligned}(\delta Q_n/\delta t)_{np}&=(\delta W_n/\delta t)_{np}-\mathbf{u}_n\cdot(\delta\rho_n\mathbf{u}_n/\delta t)_{np}\\&=-(\delta W/\delta t)_{pn}+\mathbf{u}_n\cdot(\delta\rho\mathbf{U}/\delta t)_{pn}\end{aligned} \qquad (21)$$

for neutrals, where the second line in equation (21) follows from conservation of energy and momentum in the collisions. The operators d/dt are convective derivatives following the bulk flow of the respective medium:

$$d/dt=\partial/\partial t+\mathbf{U}\cdot\nabla$$

$$(d/dt)_n=\partial/\partial t+\mathbf{u}_n\cdot\nabla.$$

$\mathbf{J}^*$ is the current density in the frame of reference of the plasma, i.e., without the advection of the charge density ρ_c,

$$\mathbf{J}^*=\mathbf{J}-\rho_c\mathbf{U}$$

and is equal to $\mathbf{J}$ for most practical purposes, by quasi-neutrality. (The distinction between the two comes from the $\rho_c\mathbf{E}$ term in the plasma momentum equation, and we need it solely for the individual-fluid approach later on.)

[13] Equations (18) and (19) are the fluid counterparts of the thermodynamic equation

$$TdS=dQ$$

and can be used to define the entropy function S in terms of pressure and density. Their right-hand sides, also invariant under change of frame of reference, can be taken as uniquely defining the heating (energy dissipation) rates. It should be noted that the heating rate is related to the rate of change of entropy and is *not* necessarily equal to the rate of change of thermal energy; the latter is given by heating *plus* work against pressure forces.

[14] Joule heating can now be identified, independently of the generalized Ohm's law, from the right-hand side of equation (18) as the electromagnetic term in the heating rate of plasma: $\mathbf{J}^*\cdot(\mathbf{E}+\mathbf{U}\times\mathbf{B}/c)$, i.e., $\mathbf{J}\cdot\mathbf{E}$ in the plasma frame of reference.

3.1. Collision Effects

[15] That the collisional heating terms $(\delta Q/\delta t)$ have the same values in all frames of reference, as claimed above, can be made manifest by expressing them in terms of the corresponding moments of the collision term $(\delta f/\delta t)_c$ in the Boltzmann equation:

$$\begin{aligned}\left(\frac{\delta Q}{\delta t}\right)_{pn}&=\int d\mathbf{v}\;\frac{1}{2}m_i|\mathbf{v}-\mathbf{U}|^2\left(\frac{\delta f}{\delta t}\right)_{pn}\\\left(\frac{\delta Q_n}{\delta t}\right)_{np}&=\int d\mathbf{v}\;\frac{1}{2}m_n|\mathbf{v}-\mathbf{u}_n|^2\left(\frac{\delta f_n}{\delta t}\right)_{np}.\end{aligned} \qquad (22)$$

Note that $(\delta Q/\delta t)_{pn}$ and $(\delta Q_n/\delta t)_{np}$ do not obey an anti-symmetry relation analogous to equation (14) for $(\delta W/\delta t)_{pn}$ and $(\delta W/\delta t)_{np}$: heat, unlike energy and momentum, is not conserved. They can, however, be rewritten, by exploiting the relations in equations (14) and (17), as each the sum of two terms common to plasma and neutrals, one term positive for both and the other with opposite signs:

$$\begin{aligned}\left(\frac{\delta Q}{\delta t}\right)_{pn}&=\left(\frac{\delta Q}{\delta t}\right)_0+\left(\frac{\delta Q}{\delta t}\right)_1\\\left(\frac{\delta Q_n}{\delta t}\right)_{np}&=\left(\frac{\delta Q}{\delta t}\right)_0-\left(\frac{\delta Q}{\delta t}\right)_1,\end{aligned} \qquad (23)$$

where

$$\left(\frac{\delta Q}{\delta t}\right)_0=\frac{1}{2}\nu_{pn}\rho|\mathbf{u}_n-\mathbf{U}|^2 \qquad (24)$$

$$\left(\frac{\delta Q}{\delta t}\right)_1=\int d\mathbf{v}\;\frac{1}{2}m_i\left|\mathbf{v}-\frac{1}{2}(\mathbf{u}_n+\mathbf{U})\right|^2\left(\frac{\delta f}{\delta t}\right)_{pn}. \qquad (25)$$

One may also rewrite the collision terms in the energy conservation equations as

$$\left(\frac{\delta W}{\delta t}\right)_{pn}=-\left(\frac{\delta W_n}{\delta t}\right)_{np}=\frac{1}{2}\nu_{in}\rho\left(u_n^2-U^2\right)+\left(\frac{\delta Q}{\delta t}\right)_1 \qquad (26)$$

(in deriving equations (24) and (26), equation (7) has been used).

[16] The term $(\delta Q/\delta t)_0$ represents a positive net heating rate, supplied equally to both plasma and neutrals, proportional to the relative bulk velocity difference; it is thus readily interpreted as heating by friction from relative motion. $(\delta Q/\delta t)_1$, by contrast, represents zero net input, with heat supplied to one medium and removed from the other; it is equal to the rate of energy change by collisions in the frame of reference in which the plasma and the neutral velocities are equal and opposite. If we assume that the collisions are elastic and that ions and neutrals are of the same species (i.e., equal masses), then, for the special case when the thermal speeds of both ions and neutrals can be neglected in comparison to $|\mathbf{u}_n-\mathbf{U}|$, evidently $(\delta Q/\delta t)_1=0$. With a little thought, it is apparent that this conclusion holds also with nonnegligible thermal energies as long as their distribution is the same for ions and for neutrals; $(\delta Q/\delta t)_1$ is nonzero only when the two media differ in thermal distribution. By dimensional arguments, we may then write

$$(\delta Q/\delta t)_1=(1/2)\nu_{pn}\rho\left(w_n^2-w^2\right)\xi, \qquad (27)$$

where w and w_n are the mean thermal speeds of plasma and neutrals, respectively, and ξ is a dimensionless number which may be a function of the dimensionless ratio

$$|\mathbf{u}_n - \mathbf{U}|^2 / \left[c_1\left(w_n^2 + w^2\right) + c_2 w_n w\right], \tag{28}$$

where c_1, c_2 are constants; ξ is expected to be of order unity, and it in any case remains finite when the ratio in equation (28) approaches either 0 or ∞.

[17] The energy equations (12) and (13) can now be written as

$$\frac{\partial}{\partial t}\left[\frac{1}{2}\rho U^2 + \epsilon\right] + \nabla \cdot \left[\mathbf{U}\left(\frac{1}{2}\rho U^2 + \epsilon\right) + \mathsf{P} \cdot \mathbf{U}\right] = \mathbf{E} \cdot \mathbf{J} + \frac{1}{2}\nu_{pn}\rho\left[u_n^2 - U^2 + \xi\left(w_n^2 - w^2\right)\right] \tag{29}$$

(plasma), and

$$\frac{\partial}{\partial t}\left[\frac{1}{2}\rho_n u_n^2 + \epsilon_n\right] + \nabla \cdot \left[\mathbf{u}_n\left(\frac{1}{2}\rho u_n^2 + \epsilon_n\right) + \mathsf{P}_n \cdot \mathbf{u}_n\right] = -\frac{1}{2}\nu_{pn}\rho\left[u_n^2 - U^2 + \xi\left(w_n^2 - w^2\right)\right] \tag{30}$$

(neutrals). The dissipation equations (18) and (19) become

$$\frac{3}{2}P\frac{d}{dt}\left[\log\left(\frac{P}{\rho^{5/3}}\right)\right] = \mathbf{J}^* \cdot \left[\mathbf{E} + \frac{\mathbf{U}}{c} \times \mathbf{B}\right] + \frac{1}{2}\nu_{pn}\rho\left[|\mathbf{u}_n - \mathbf{U}|^2 + \xi\left(w_n^2 - w^2\right)\right] \tag{31}$$

(plasma), and

$$\frac{3}{2}P_n\left(\frac{d}{dt}\right)_n\left[\log\left(\frac{P_n}{\rho_n{}^{5/3}}\right)\right] = +\frac{1}{2}\nu_{pn}\rho\left[|\mathbf{u}_n - \mathbf{U}|^2 - \xi\left(w_n^2 - w^2\right)\right] \tag{32}$$

(neutrals), independent of frame of reference as they should be.

4. Application to the Ionosphere/Thermosphere System

[18] At ionospheric heights, several different gases coexist in the same volume of space: the plasma and the neutral atmosphere, each in turn containing several different species, interacting through collisions between charged and neutral particles. We have mostly ignored the multineutral and the multi-ion aspects in this work, but for plasma one must always include at least the electrons as a species different from the ions. That the bulk velocities of electrons and ions differ is implied by the presence of electric current. The plasma as a whole and the neutral atmosphere also have, in general, different bulk velocities; this fact underlies the ambiguities of defining an Ohm's law for the ionosphere, as discussed earlier.

4.1. Plasma and Neutral Atmosphere as Two Separate Fluids

[19] The most common approach in dealing with the electrodynamics and energetics of the ionospheric regions is to treat the plasma and the neutral particles as two separate fluids, each with its own density, bulk velocity, and pressure. The equations derived in the previous section are directly applicable. Equations (29) and (30) are the energy balance equations; they can be added together to get the energy balance of the whole medium (plasma together with neutrals). The only net source of energy is $\mathbf{J} \cdot \mathbf{E}$, electromagnetic energy supplied to the plasma; in addition, there is energy exchange between plasma and neutrals, an equal amount being taken from one and added to the other by collisions. Equations (31) and (32) are the dissipation equations; in general they cannot be added to each other or to the energy balance equations. Energy is dissipated in the plasma by $\mathbf{J}^* \cdot (\mathbf{E} + \mathbf{U} \times \mathbf{B}/c)$, which is Joule heating as properly defined ($\mathbf{J} \cdot \mathbf{E}$ in the plasma frame of reference) and, according to the discussion following equation (10), generally is small except at the lowest altitudes; in addition, energy is dissipated by collisions, the relative flow between plasma and neutrals contributing the same positive amount to both, and the difference between plasma and neutral temperatures contributing equal amounts of opposite sign. (Note that the expression $\mathbf{J} \cdot (\mathbf{E} + \mathbf{u}_n \times \mathbf{B}/c)$, Joule heating as conventionally defined by $\mathbf{J} \cdot \mathbf{E}$ in the neutral frame of reference, does not appear anywhere.)

4.2. Plasma and Neutral Atmosphere as One Fluid

[20] As an alternative, it is possible to treat the plasma and the neutral medium as a single fluid, with total mass density ρ_T given by the sum of plasma and neutral components and total bulk velocity $\mathbf{U}_T$ defined by the linear momentum of the entire medium:

$$\rho_T = \rho + \rho_n \qquad \rho_T \mathbf{U}_T = \rho \mathbf{U} + \rho_n \mathbf{u}_n, \tag{33}$$

hence

$$\mathbf{U}_T = (\alpha \mathbf{U} + \mathbf{u}_n)/(1 + \alpha), \tag{34}$$

where $\alpha \equiv \rho/\rho_n$. With this approach there are no collision terms in the momentum and energy equations for the medium as a whole (as long as collisions conserve energy and momentum and are nonionizing). The energy conservation equation is

$$\frac{\partial}{\partial t}\left[\frac{1}{2}\rho_T U_T^2 + \epsilon_T\right] + \nabla \cdot \left[\mathbf{U}_T\left(\frac{1}{2}\rho_T U_T^2 + \epsilon_T\right) + \mathsf{P}_T \cdot \mathbf{U}_T\right] = \mathbf{E} \cdot \mathbf{J} - \nabla \cdot \mathbf{q}_T \tag{35}$$

Equation (35) is the sum of equations (12) and (13), which together with the definitions (33) and (34) yields the expressions for the thermal quantities of the medium as a whole:

$$\epsilon_T = \epsilon + \epsilon_n + \frac{\rho}{2(1+\alpha)}|\mathbf{u}_n - \mathbf{U}|^2 \tag{36}$$

$$\mathsf{P}_T = \mathsf{P} + \mathsf{P}_n + \frac{\rho}{1+\alpha}(\mathbf{u}_n - \mathbf{U})(\mathbf{u}_n - \mathbf{U}) \tag{37}$$

$$\mathbf{q}_T = \frac{\mathbf{u}_n - \mathbf{U}}{1+\alpha}\left(\alpha\epsilon_n - \epsilon - \frac{\rho(1-\alpha)}{2(1+\alpha)}|\mathbf{u}_n - \mathbf{U}|^2\right) + \frac{\mathbf{u}_n - \mathbf{U}}{1+\alpha} \cdot (\alpha \mathsf{P}_n - \mathsf{P}). \tag{38}$$

Whenever there is a bulk velocity difference between plasma and neutrals, $\mathbf{u}_n - \mathbf{U} \neq 0$, the combined plasma-neutral medium has a "thermal" energy density that includes a contribution from kinetic energy of relative bulk motion, and it also has a nonisotropic pressure and a nonzero heat flux even if these are absent both in the plasma and in the neutral medium considered by themselves. (The unavoidable presence of these terms must be counted as a disadvantage of the single-fluid approach, offsetting the advantage of having no collision terms.) The dissipation equation is

$$\frac{3}{2}P_T\left(\frac{d}{dt}\right)_T\left[\log\left(\frac{P_T}{\rho_T^{5/3}}\right)\right] = \mathbf{J}^* \cdot \left[\mathbf{E} + \frac{\mathbf{U}_T}{c} \times \mathbf{B}\right] - \nabla \cdot \mathbf{q}_T - \sum_{lk} \mathcal{S}_{lk} \frac{\partial (U_T)_l}{\partial r_k}, \qquad (39)$$

where now, of course,

$$(d/dt)_T = \partial/\partial t + \mathbf{U}_T \cdot \nabla$$

and

$$\mathbf{J}^* = \mathbf{J} - \rho_c \mathbf{U}_T.$$

$\mathcal{S}$ is the traceless part of the pressure tensor

$$P_T = P_T I + \mathcal{S}$$

with $P_T = (1/3)(\mathrm{Trace}\ P_T)$, and the entire last term represents viscous heating, which has been included together with the heat flux term, in accordance with the discussion above. The electromagnetic dissipation in equation (39) is $\mathbf{J}^* \cdot (\mathbf{E} + \mathbf{U}_T \times \mathbf{B}/c)$, i.e., $\mathbf{J} \cdot \mathbf{E}$ in the frame of reference of the total medium, which is close to the conventional value $\mathbf{J} \cdot (\mathbf{E} + \mathbf{u}_n \times \mathbf{B}/c)$ numerically when $\alpha \ll 1$ but different conceptually; furthermore, the meaning of "heating" is now ambiguous, kinetic energy of relative bulk motion counting as part of thermal energy.

4.3. Each Plasma Component as Separate Fluid

[21] At the other extreme, it is possible to treat, in addition to the neutral medium, each charged particle species, including the electrons, as a separate fluid with its own density, bulk velocity, and pressure. The energy balance and the energy dissipation equations can be derived for each fluid separately, by manipulating the moment equations as before but without summing over species. The resulting equations are identical in form to those previously derived, with each quantity referring to species a and with the replacements

$$\mathbf{J} \rightarrow q_a n_a \mathbf{V}_a \qquad \rho_c \rightarrow q_a n_a,$$

which immediately implies that $\mathbf{J}^* \rightarrow 0$ for all a. In this approach, therefore, for any individual species, all electromagnetic energy input represents work done—there is no electromagnetic heating. Electromagnetic dissipation thus appears only as a consequence of interactions among charged-particle species, of which there must therefore be more than one (at the very least, electrons and one species of positive ions).

5. Assumptions Underlying the Conventional View

[22] In none of the general statements of the energy conservation and dissipation equations, as formulated here with several alternative approaches, does the conventionally used expression for ionospheric Joule heating, $\mathbf{J} \cdot \mathbf{E}$ in the frame of reference of the neutral atmosphere, appear as a significant quantity. Any basis for the applicability of this expression must lie therefore not in the general formulation but in some specific assumption.

[23] Let us add together the heating rates of plasma and of neutrals, designating by $\mathcal{Q}$ the sum of the right-hand sides of the plasma and neutral dissipation equations (31) and (32). (It should be noted that the physical meaning of such a sum is not entirely clear; see discussion following equation (45)). Neglecting the difference between $\mathbf{J}^*$ and $\mathbf{J}$,

$$\mathcal{Q} = \mathbf{J} \cdot \left[\mathbf{E} + \frac{\mathbf{U}}{c} \times \mathbf{B}\right] + \nu_{pn}\rho|\mathbf{u}_n - \mathbf{U}|^2. \qquad (40)$$

With the use of equations (7) and (4) we then obtain

$$\mathbf{J} \cdot \left[\mathbf{E} + \frac{\mathbf{u}_n}{c} \times \mathbf{B}\right] = \mathcal{Q} + (\mathbf{U} - \mathbf{u}_n) \cdot \left[\frac{\partial \rho \mathbf{U}}{\partial t} + \nabla \cdot \boldsymbol{\kappa}\right]. \qquad (41)$$

The expression on the right-hand side is a hybrid: a heating term plus a term that equals the work done on the plasma in the frame of reference of the neutrals. This term can be dropped under the assumption (already mentioned in our discussion of $\mathbf{J} - \mathbf{E}$ relations as the basis for the conventional formulation of ionospheric Ohm's law) that all terms on the left-hand side of the plasma momentum equation (4) are negligible, reducing the equation to

$$0 = \mathbf{J} \times \mathbf{B}/c + (\delta \rho \mathbf{U}/\delta t)_{pn}, \qquad (42)$$

that is, the linear momentum supplied by the magnetic stress on the plasma is assumed to be entirely transferred by collisions to the neutral atmosphere, with negligible transfer to the plasma itself. Then the conventional expression for the ionospheric Joule heating becomes

$$\mathbf{J} \cdot \left[\mathbf{E} + \frac{\mathbf{u}_n}{c} \times \mathbf{B}\right] = \mathcal{Q}, \qquad (43)$$

which does indeed represent the complete combined heating rate of plasma plus neutrals. That heating rate, however, is the sum of the true Joule heating (in the plasma frame of reference) $\mathbf{J} \cdot (\mathbf{E} + \mathbf{U} \times \mathbf{B}/c)$ and of twice the common collisional heating term $(\delta Q/\delta t)_0$ (equation (24)), once for plasma and once for neutrals. As noted before, the true Joule heating depends on electron collisions (equation (11)) and is small over much of the ionosphere. The term $(\delta Q/\delta t)_0$ comes from collisional effects in the dissipation equations, associated with relative bulk flow and entirely independent of $\mathbf{E}$ or $\mathbf{J}$ (other than that these happen to be associated with the existence of the bulk flow difference between plasma and neutrals). What is conventionally called ionospheric Joule heating is thus primarily frictional heating—mechanical and not electromagnetic dissipation.

[24] Applied to the kinetic energy equations (15) and (16), the assumption expressed by equation (42) gives zero on the right-hand side of the plasma equation. This is simply a restatement of the assumption and has no further physical meaning (in particular, it does *not* imply that temporal changes of plasma flow are due to pressure effects alone!). The equation for neutrals may be rewritten as

$$\frac{\partial}{\partial t}\left[\frac{1}{2}\rho_n u_n^2\right] + \nabla \cdot \left[\mathbf{u}_n\left(\frac{1}{2}\rho u_n^2\right)\right] + [\nabla \cdot \mathsf{P}_n] \cdot \mathbf{u}_n = \mathbf{J} \cdot \left[\frac{\mathbf{u}_n}{c} \times \mathbf{B}\right], \tag{44}$$

a mathematical restatement of work done by collisions as electromagnetic. As discussed above, whether this work changes the flow or the thermal energy of the neutrals cannot be determined without detailed calculation. Only if the spatial gradients are assumed negligible does all the work go into increasing the neutral bulk-flow kinetic energy; this assumption is frequently made, with the argument that only large-scale horizontal flows are involved, but there may exist viscosity effects (nonisotropic P_n) which can couple horizontal flows with the (steep) vertical gradients (an example is discussed by *Song et al.* [2004]).

5.1. Neutral-Wind Versus Center-of-Mass Frame

[25] As long as most of the mass density is in the neutrals, $\rho/\rho_n \equiv \alpha \ll 1$, the bulk velocity $\mathbf{u}_n$ of the neutral medium is nearly the same as $\mathbf{U}_T$, the bulk velocity of the entire medium plasma-plus-neutrals (sometimes referred to as the center-of-mass velocity). The conventional Joule heating should then also be nearly the same if calculated in the frame of reference $\mathbf{U}_T$ instead of $\mathbf{u}_n$. From equations (43), (42), and (7) it is easily shown that

$$\begin{aligned}\mathbf{J} \cdot \left[\mathbf{E} + \frac{\mathbf{U}_T}{c} \times \mathbf{B}\right] &= \mathcal{Q} + \frac{1}{c}\mathbf{J} \times \mathbf{B} \cdot (\mathbf{u}_n - \mathbf{U}_T) \\ &= \mathcal{Q} - \frac{\alpha}{1+\alpha}\nu_{pn}\rho|\mathbf{u}_n - \mathbf{U}|^2,\end{aligned} \tag{45}$$

which, as expected, differs negligibly from equation (43) if $\alpha \ll 1$. Note that the small difference is always negative: the heating rate in equation (45) (using the center-of-mass, one-fluid bulk velocity) is smaller than the heating rate (43) (shown to equal the sum of the separate plasma and neutral heating rates); by contrast, the thermal energy density in the one-fluid formulation (36) is larger than the sum of plasma and neutral thermal energy densities, because it now also includes kinetic energy of relative bulk motion. As indicated in the first line of equation (45), the difference is simply the work done on the neutrals in the center-of-mass frame. Although negligible in practice as noted already, it suggests that there may be some ambiguity about distinguishing heating and work done in the case of ad hoc constructions such as equation (40).

5.2. Implications for the Neutral Atmosphere

[26] The consequences of the assumption expressed by equation (42) can be seen more generally if we restate the momentum equations, in standard conservation form, for both plasma and the neutral medium:

$$\partial\rho\mathbf{U}/\partial t + \nabla \cdot \boldsymbol{\kappa} = \mathbf{J} \times \mathbf{B}/c + (\delta\rho\mathbf{U}/\delta t)_{pn}$$

$$\partial\rho_n\mathbf{u}_n/\partial t + \nabla \cdot \boldsymbol{\kappa}_n = (\delta\rho_n\mathbf{u}_n/\delta t)_{np} = -(\delta\rho\mathbf{U}/\delta t)_{pn}.$$

In each equation, the right-hand side represents the forces acting on the respective medium; the left-hand side represents the momentum transfer and contains only time and space derivatives. The assumption that leads to equation (42) is of course just the neglect of the left-hand side of the plasma equation, possible because the right-hand side contains *two* terms, which can balance each other and each of which by itself can be large in comparison to the left-hand side. By contrast, the right-hand side of the neutral atmosphere equation contains only *one* term, and hence the left-hand side can *never* be neglected. Furthermore, as long as the assumption (42) holds, the one term on the right-hand side of the neutral atmosphere equation equals the $\mathbf{J} \times \mathbf{B}/c$ force. Thus somewhat paradoxically perhaps, the electromagnetic stresses exerted from the magnetosphere on the ionosphere must be balanced by nonzero time and/or space derivatives of the neutral medium, while the corresponding derivatives of the plasma are assumed negligible.

6. Summary and Conclusions

[27] The electric current density in the ionosphere is usually regarded as driven by the electric field in the frame of reference of the neutral atmosphere, with resistivity arising primarily from ion-neutral collisions. As shown by *Song et al.* [2001], however, it can equally well be regarded as driven by the electric field in the frame of reference of the plasma, with a much smaller resistivity due almost entirely to electron collisions only. There is a corresponding ambiguity about how ionospheric Joule heating should be calculated. Using the rigorous generalized Ohm's law, we have shown that a current driven by the electric field in the strict sense (i.e., an applied $\mathbf{E}$ produces a $\mathbf{J}$ that increases until it is limited by collisions) is proportional to $\mathbf{E}$ in the frame of reference of the plasma bulk flow and depends on electron collisions. The ionospheric current as conventionally described, proportional to $\mathbf{E}$ in the frame of reference of the neutral atmosphere and depending on ion-neutral collisions, comes not from the generalized Ohm's law but from the plasma momentum equation: it is the current of which the $\mathbf{J} \times \mathbf{B}/c$ force balances the frictional force from the relative bulk motion between plasma and neutrals.

[28] We have analyzed the energy equations both for plasma and for neutrals, taking collisions into account, in two ways: (1) conservation equations for total energy, valid in any frame of reference, show a net input $\mathbf{J} \cdot \mathbf{E}$ of electromagnetic energy into the plasma, together with energy transfer (no net input) by collisions between plasma and neutrals, all the amounts depending on frame of reference; (2) heating/dissipation equations, derived by subtracting the work done and independent of frame of reference, show Joule heating $\mathbf{J} \cdot (\mathbf{E} + \mathbf{U} \times \mathbf{B}/c)$ of plasma, plus an equal amount of heating by collisions of plasma and

of neutrals, plus some heat transfer between plasma and neutrals if their temperatures differ. All these heating rates add up to the quantity $\mathbf{J} \cdot (\mathbf{E} + \mathbf{u}_n \times \mathbf{B}/c)$, the conventional expression for ionospheric Joule heating, which thus would seem to be more accurately describable as heating by friction between plasma and neutrals. If the plasma and the neutral atmosphere are treated as one fluid with one bulk velocity (instead of as two fluids, each with its own bulk velocity), with the relative velocity between plasma and neutrals then counting as "thermal" velocity, a slightly (but not significantly) different heating rate is obtained, which describes the heat input into the entire medium without distinguishing between plasma and neutrals.

[29] The main conclusions from our work are the following:

[30] 1. Joule heating is properly defined in the plasma frame of reference. The "ionospheric Joule heating" as conventionally defined is not primarily Ohmic or Joule heating in the physical sense but is for the most part simply frictional heating from the relative motion of plasma and neutrals. That it has the mathematical form of Joule heating is largely a coincidence (resulting from the assumption discussed in conclusion 3 below): the current density $\mathbf{J}$ is related physically to $\mathbf{U} - \mathbf{u_n}$ by the plasma momentum equation, and $\mathbf{U} - \mathbf{u_n}$ is related to $\mathbf{E}$ by the generalized Ohm's law, but there is no direct physical relation between $\mathbf{J}$ and $\mathbf{E}$ (unlike Ohm's law in ordinary conducting materials).

[31] 2. The energy equation can be formulated in several ways, in terms either of total energy input, of work done, or of heating, but choosing among the different forms of the equation is not the same as separating mechanical energy into bulk-flow kinetic or thermal energy. The common assumption that (1) $\mathbf{u}_n \cdot \mathbf{J} \times \mathbf{B}/c$ goes into flow and (2) heating goes into thermal energy is in general not justified; partitioning into flow or thermal energy depends for case 1 on whether the inertial or the pressure terms balance the force and for case 2 on whether the heating or the work done against pressure is more important.

[32] 3. A crucial assumption both for the ionospheric Ohm's law and for the conventional formulation of ionospheric Joule heating is that, to a good approximation, the $\mathbf{J} \times \mathbf{B}/c$ force equals the drag force between plasma and neutral flow. There is no particular reason to question the validity of the assumption, but it does imply that, to the same degree of approximation, all the electromagnetic stresses exerted from the magnetosphere on the ionosphere must be balanced by the neutral atmosphere. Magnetosphere-ionosphere interactions thus always involve a dynamical response of the neutral atmosphere, a response that should not be neglected unless there is an adequate justification for doing so.

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